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Inflation with bulk fields in the Randall-Sundrum warped compactification?

J. M. Cline
McGill University, 3600 University St. Montréal, Québec H3A 2T8, Canada
E-mail: jcline@physics.mcgill.ca

U. A. Yajnik
Indian Institute of Technology Bombay, Mumbai 400 076, India
E-mail: yajnik@phy.iitb.ernet.in

Abstract. Randall and Sundrum have proposed that we live on a brane embedded in 5-dimensional anti-deSitter space, as a solution to the hierarchy problem. We examine the possibility of using a scalar field in the 5-D bulk as the inflaton, and show that it gives results which are indistinguishable from an inflaton which is restricted to our brane. This makes it difficult to get large enough density perturbations in simple one-field inflationary models, since mass scales on our brane are supposed to be limited by a cutoff of order 1 TeV.

Keywords. Inflation, Extra Dimensions

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1. Introduction

The Randall-Sundrum proposal for solving the hierarchy problem has received much attention in the last year [1]. They considered an extra compact dimension with coordinate y and line element

$$ds^2 = a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + b^2 dy^2 \tag{1}$$

where $a(y) = e^{-kb|y|}$, $y \in [-1, 1]$ and the points y and $-y$ are identified so the extra dimension is an orbifold with fixed points at $y = 0$ and $y = 1$. The four-dimensional geometry is conformal to Minkowski space. The scale k in the warp factor $a(y)$ is determined by the 5-D cosmological constant, Λ and the analog to the Planck mass, M, by $k = (-\Lambda/6M^3)^{1/2}$. Hence Λ must be negative, and the 5-D space is anti-deSitter.

At $y = 0$ there is a positive tension brane (the Planck brane) on which particle masses are naturally of order the Planck mass, M_p, while at $y = 1$ there is a negative tension brane (called the TeV brane), where particle masses are suppressed by the

warp factor e^{-kb}. By adjusting the size of the extra dimension, b, so that $kb \sim 37$, the masses of particles on the TeV brane will be in the TeV range, even if all the underlying mass parameters (including Λ, M and k) are of order M_p to the appropriate power.

Although this is desirable for solving the hierarchy problem, it makes it difficult to understand the origin of inflation. The density perturbations from inflation, $\delta\rho/\rho$, are suppressed by inverse powers of M_p. To get $\delta\rho/\rho \sim 10^{-5}$, one needs a mass scale much larger than 1 TeV in the numerator. By construction, the TeV scale is the cutoff on the TeV brane, so it is hard to see where such a scale could come from unless some physics outside of the TeV brane is invoked.

2. Inflation with a bulk scalar

In this study we investigate what happens when the inflaton is a bulk scalar field. The simplest possibility is chaotic inflation with a free field [2]. We will assume that the line element (1) is modified by replacing the Minkowski metric with 4-D deSitter space, whose line element is $ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$. The action for the bulk scalar is then

$$S = \frac{1}{2}\int d^4x\, dy\, b\, e^{3Ht} a^4(y) \left[a^{-2}(y)\dot{\phi}^2 - b^{-2}\phi'^2 - m^2\phi^2\right] \tag{2}$$

The equation of motion for ϕ is

$$a^{-2}\left(\ddot{\phi} + 3H\dot{\phi}\right) - b^{-2}\left(\phi'' - 4kb\phi'\right) + m^2\phi = 0 \tag{3}$$

and, assuming that the size of the extra dimension is stabilized [3], the Hubble rate is given by

$$H^2 = \frac{4\pi G}{3}\int_0^1 dy\, b\, a^4(y)\left(a^{-2}\dot{\phi}^2 + b^{-2}\phi'^2 + m^2\phi^2\right), \tag{4}$$

where G is the ordinary 4-D Newton's constant.

We look for a separable solution, $\phi = \phi_0(t) f(y)$. We take ϕ_0 to have dimensions of $(\text{mass})^1$ so that it is the canonically normalized field in an effective 4-D description, and f has dimensions of $(\text{mass})^{1/2}$. We will also assume the slow roll condition is fulfilled so that the terms $\ddot{\phi}$ and $\dot{\phi}^2$ can be ignored in the last two equations. The equation of motion becomes

$$\dot{\phi}_0 = \frac{e^{-2kby}}{3\hat{H}}\left(-m^2 + \frac{1}{b^2 f}\left(f'' - 4kbf'\right)\right) = \text{constant} \equiv -\Omega \tag{5}$$

with

$$\hat{H}^2 \equiv \frac{H^2}{\phi_0^2} = \frac{4\pi G}{3}\int_0^1 dy\, b\, e^{-4kby}\left(b^{-2}f'^2 + m^2 f^2\right). \tag{6}$$

The solution for ϕ_0 is obviously linear, $\phi_0(t) = C - \Omega t$. For chaotic inflation we want $C \gg M_p$ (necessary to fulfill the slow roll condition [2]) and $\Omega > 0$, so that ϕ_0 is rolling to the minimum of its potential.

The equation for f becomes

$$f'' - 4kbf' - b^2 \left(m^2 - 3\hat{H}\Omega e^{2kby}\right) f = 0, \tag{7}$$

This is the same equation as (7) of ref. [4]. The solutions f_n (called y_n in [4]) are discrete, such that $\hat{H}\Omega$ is quantized:

$$3k^{-2}\hat{H}\Omega e^{2kb} = x_{n\nu}^2 \tag{8}$$

Here $x_{n\nu}$ is the nth root of the equation $2J_\nu(x_{n\nu}) + x_{n\nu}J'_\nu(x_{n\nu}) = 0$, where the order of the Bessel function is $\nu = \sqrt{4 + m^2/k^2}$. For m/k in the range $0.5 - 3$, the lowest mode $x_{1,\nu}$ ranges from 4 to 6. This assumes the boundary condition that $f' = 0$ at $y = 0, 1$, but other choices of boundary conditions will lead to essentially identical conclusions, as we will explain below. The f_n's are normalized so that

$$\int_0^1 dy\, b\, e^{-2kby} f_n(y) f_m(y) = \delta_{mn} \tag{9}$$

With the solution for f_n we can evaluate the rescaled Hubble rate, $\hat{H}$, in eq. (6). After a partial integration and use of the equation of motion (7), the integral in (6) becomes identical to that of (9), times $3\hat{H}\Omega$. This gives $\hat{H}^2 = 4\pi G\hat{H}\Omega$, which together with eq. (8) determines Ω, the rate at which ϕ_0 is rolling to its minimum:

$$\Omega = \frac{k x_{n\nu} e^{-kb}}{\sqrt{12\pi G}} \sim M_p \times 1\text{TeV} \tag{10}$$

We used the fact that e^{-kb} is supposed to be of order (TeV)$/M_p$.

3. Density perturbations

We can now estimate the magnitude of density perturbations in this model. Using $\delta\rho/\rho \sim H^2/|\dot{\phi}_0|$,

$$\frac{\delta\rho}{\rho} \sim (4\pi G)^2\, \Omega\, (C - \Omega t)^2 \sim \frac{\text{TeV}}{M_p} \tag{11}$$

Although C is presumed to be super-Planckian, $(C - \Omega t)$ will not be orders of magnitude larger than M_p near the end of inflation, when the perturbations with COBE-scale wavelengths were being produced; hence we take $(C - \Omega t) \sim M_p$ in the above estimate. This suppression of the density perturbations makes our model not viable.

Nevertheless, it is interesting to compare to what would happen if we tried to do chaotic inflation using a scalar field trapped on the TeV brane. The mass of

the field is now constrained to be of order $m \sim$ TeV because of the suppression of masses by the warp factor. The equation of motion during the slow roll regime is

$$\dot{\phi} = -\frac{m^2\phi}{3H} = -\frac{m^2\phi}{\sqrt{4\pi G m^2\phi^2/3}}; \tag{12}$$

hence ϕ evolves linearly with time, and $\dot{\phi}$ is of order $M_p \times 1$TeV, just as with the bulk scalar field. And the estimate for $\delta\rho/\rho$ has the same parametric form. All this, despite the fact that we started with a bulk scalar whose mass is Planck-scale in the 5-D Lagrangian.

In retrospect, this result is not surprising. Reference [4] noted that the modes of the bulk scalar behave similarly to TeV-scale particles on the brane. This can be understood by the form of the solutions [4],

$$f_n \sim e^{2ky} J_\nu \left(x_{n\nu} e^{kb(y-1)} \right), \tag{13}$$

which are strongly peaked near $y = 1$. There is thus little practical difference between the low-lying modes of the bulk field and a field confined to the TeV brane.

One might wonder if this conclusion depends on the choice of boundary conditions for the modes f_n. However the fact that the modes peak at the TeV brane comes from bulk energetics, not boundary conditions. Since the mass of the bulk field is effectively varying like e^{-kby}, it is energetically much more efficient for the field to be concentrated near $y = 1$.

4. Conclusion

The simplest chaotic inflation models seem to be ruled out in the Randall-Sundrum scenario, whether the inflaton is a bulk field or one restricted to the TeV brane. One could, alternatively, put the inflaton on the Planck brane (at $y = 0$), at the cost of reintroducing a hierarchy problem—why should m/M_P be $O(\delta\rho/\rho) \sim 10^{-5}$? This fine-tuning problem always occurs in inflation, but the RS setting casts it in a somewhat new light. One could invent an intermediate brane for the inflaton, which has just the right mass scale, but this seems artificial. Perhaps the RS idea, if correct, is telling us that hybrid inflation (involving more than one field) is necessary.

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